Frequency Space Environment Map Rendering

Ravi Ramamoorthi
ravir@graphics.stanford.edu

Pat Hanrahan
hanrahan@graphics.stanford.edu

Stanford University

http://graphics.stanford.edu/papers/freqenv/

Demo

Motivation: Interactive rendering with complex natural illumination and realistic, measured BRDFs

Reflection Equation

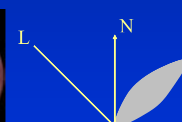
$$L\left(R(\vec{N})\,\vec{l}\right)$$

2D Environment Map

Reflection Equation

$$L\left(R(\vec{N})\,\vec{l}\right)\,\rho\left(\vec{l},\vec{V}\right)$$

2D Environment Map BRDF



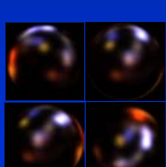
Reflection Equation

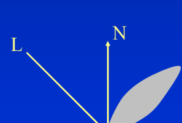
$$B(\vec{N},\vec{V}) = \int_{\Omega} L\left(R(\vec{N})\,\vec{l}\right)\,\rho\left(\vec{l},\vec{V}\right)\,dl$$

4D Orientation Light Field

2D Environment Map

BRDF





Previous Work: Blinn & Newell 76, Miller & Hoffman 84, Greene 86, Kautz & McCool 99, Cabral et al. 99, …

Goals

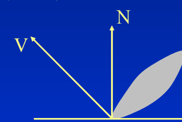
- Efficiently precompute and represent OLF
- Real-time rendering with OLF

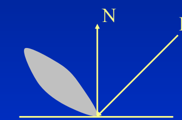
Questions

- Parameterization and structure of OLF
- Structure leads to representation
- Computation and rendering of OLF

OLF Parameterization

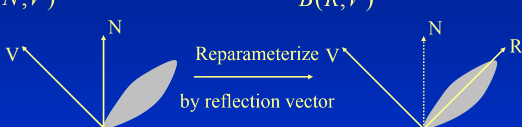
$B(\vec{N},\vec{V})$





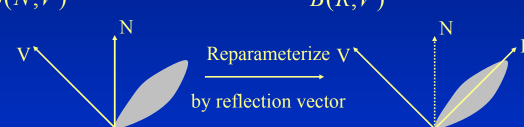
OLF Parameterization

$B(\vec{N},\vec{V})$ $\qquad$ $B(\vec{R},\vec{V})$



OLF Parameterization

$B(\vec{N},\vec{V})$ $\qquad$ $B(\vec{R},\vec{V})$



- Captures structure of BRDF (and hence OLF) better
- Reflective BRDFs become low-dimensional

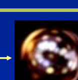
OLF Structure

$$B(\vec{R},\vec{V})$$

$B_V(\vec{R})$
2D view array of reflection maps

$B_R(\vec{V})$
2D image array of view maps

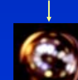
OLF Structure: Phong

Environment Map  →  Phong Reflection Map (blurred environment map)

$$B(\vec{R},\vec{V}) = B(\vec{R})$$

$B_V(\vec{R})$
2D view array of reflection maps

$B_R(\vec{V})$
2D image array of view maps

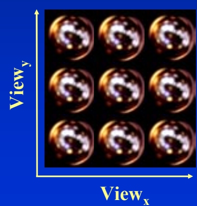


Same reflection map for all views

OLF Structure: Phong

$$B(\vec{R},\vec{V}) = B(\vec{R})$$

$B_V(\vec{R})$ $B_R(\vec{V})$

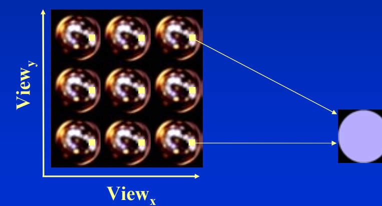


Same reflection map for all views

OLF Structure: Phong

$$B(\vec{R},\vec{V}) = B(\vec{R})$$

$B_V(\vec{R})$ $B_R(\vec{V})$



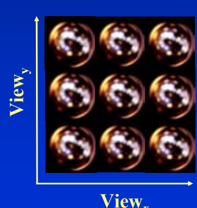
Same reflection map for all views

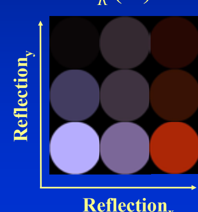
View maps constant for each R

OLF Structure: Phong

$$B(\vec{R},\vec{V}) = B(\vec{R})$$

$B_V(\vec{R})$ $B_R(\vec{V})$



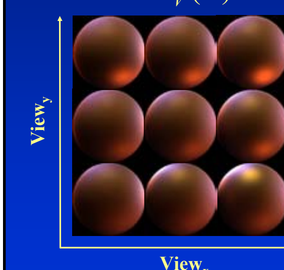


Same reflection map for all views

View maps constant for each R

OLF Structure: Lafortune

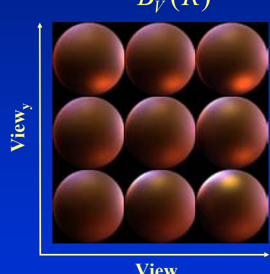
$B_V(\vec{R})$



- Single 2D reflection map no longer sufficient
- But variation with viewing direction is slow

OLF Structure: Lafortune

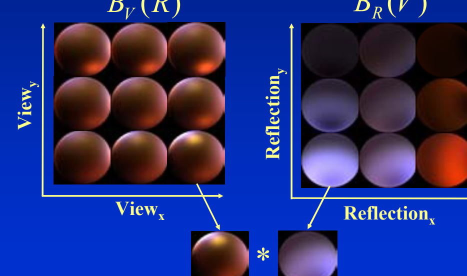
$B_V(\vec{R})$ $B_R(\vec{V})$



View maps vary slowly

A Simple Factorization

$B_V(\vec{R})$ $B_R(\vec{V})$



$$B(\vec{R},\vec{V}) \approx f(\vec{R}) * g(\vec{V})$$

Questions

- Parameterization and structure of OLF
- Structure leads to representation
  - Frequency space analysis
- Computation and rendering of OLF

Convolution

$$B(\vec{N},\vec{V}) = \int_{\Omega} L\left(R(\vec{N})\,\vec{l}\right)\,\rho\left(\vec{l},\vec{V}\right)\,dl$$

$$B = L \otimes \rho$$

Spherical harmonic analysis

$$B_{ij} = L_i \rho_{ij}$$

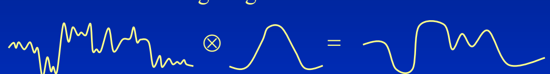
Spatial: integral

Frequency: product

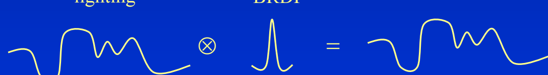
Ramamoorthi and Hanrahan 01

Implications

- Information content of OLF determined by information in lighting and BRDF



High frequency lighting ⊗ Low frequency BRDF = Low frequency OLF



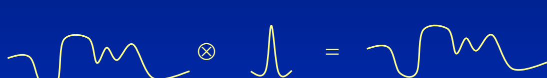
Low frequency lighting ⊗ High frequency BRDF = Low frequency OLF

Implications

Sampling rates/resolutions

- Minimum of highest light, BRDF frequencies
- Angular resolution proportional to max frequency

Example: Low frequency L



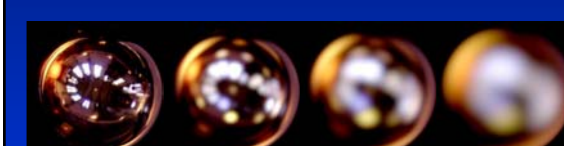
Low frequency lighting ⊗ High frequency BRDF = Low frequency OLF

Example: Low frequency lighting [Sloan et al. 02]

- OLF is low frequency
- Represent with low-order spherical harmonics only
- Compute OLF using coefficient multiply [Cabral et al. 87, Kautz et al. 02]

Natural Lighting

Natural (high frequency) lighting



4000 terms | 400 terms | 100 terms | 36 terms

Hybrid Representation

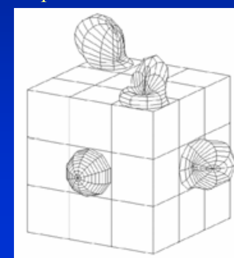
- Reflection maps $B_V(\vec{R})$ are high frequency

- View maps $B_R(\vec{V})$ are low frequency

- Use hybrid angular frequency-space representation
  - View maps: Use low-order spherical harmonic expansion
  - Represent coefficient reflection maps explicitly

Spherical Harmonic Reflection Map

- View-dependent reflection (cube)map
- Encode view maps $B_R(\vec{V})$ with low-order spherical harmonics

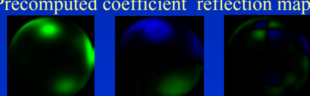


Spherical Harmonic Reflection Map

$$B(\vec{R},\vec{V}) = \sum_{i=0}^{N} B_i(\vec{R})\, Y_i(\vec{V})$$

Spherical Harmonics

Precomputed coefficient reflection maps



B_0 B_1 B_2

Questions

- Parameterization and structure of OLF
- Structure leads to representation
- Computation and rendering of OLF

Prefiltering

$$L,\rho \longrightarrow \boxed{L_i,\rho_{ij} \longrightarrow B_{ij} = L_i\rho_{ij}} \longrightarrow B_i(\vec{R})$$

Input Lighting and BRDF — Spherical Harmonic coeffs. — Convolution — SHRM

- Directly compute SHRM from Lighting, BRDF
- Convolution easier to compute in frequency domain

Prefiltering

$$L,\rho \longrightarrow \boxed{L_i,\rho_{ij} \longrightarrow B_{ij} = L_i\rho_{ij}} \longrightarrow B_i(\vec{R})$$

Spherical Harmonic coeffs. — Convolution

- 3 to 4 orders of magnitude faster (< 1 s compared to minutes or hours)
- Detailed analysis, algorithms, experiments in paper

SHRM Rendering

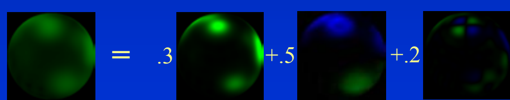
We create dynamic reflection map per frame

- Weighted sum of prefiltered coefficient reflection maps

$$B_V(\vec{R}) = \sum_{i=0}^{N} Y_i(\vec{V})\, B_i(\vec{R})$$

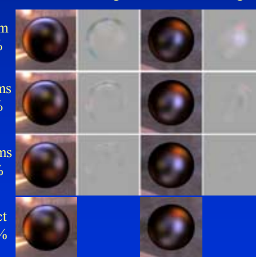
Spherical Harmonics (fixed weighting factor)

Prefiltered coefficient reflection maps



$Y_0 \quad B_0 \qquad Y_1 \quad B_1 \qquad Y_2 \quad B_2$

Number of SHRM terms



Number of terms: CUReT

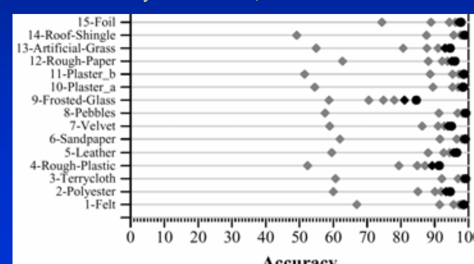
CUReT database [Dana et al 99]

- 61 BRDFs of real materials
  205 measurements for each
- Interpolated using order 8
  Zernike polynomials
  [Koenderink, van Doorn 98]

Number of terms: CURET

Analysis for all 61 samples [full bar chart in paper]

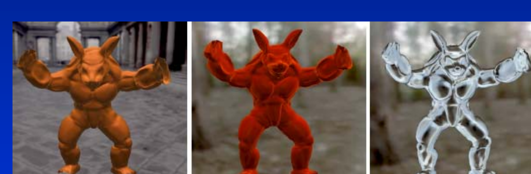
- For essentially all materials, 9-16 terms in SHRM suffice



Implementation

- Stanford Real-Time Programmable Shading System
- SHRMs used in any shader just like reflection map
- New reflection map computed for each frame
- Real-time (>15Hz) performance on 1.4 GHz Pentium IV with nVidia Geforce 2
- http://graphics.stanford.edu/papers/freqenv/

Demo



Summary of Contributions

- Theoretical, empirical analysis of sampling rates and resolutions
  - Frequency space analysis directly on lighting, BRDF
  - Low order expansion suffices for essentially all BRDFs
- Spherical Harmonic Reflection Maps
  - Hybrid angular-frequency space
  - Compact, efficient, accurate
  - Easy to analyze errors, determine number of terms
- Fast computation using convolution

Implications and Future Work

- Frequency space methods for rendering
  - Global illumination
  - Fast computation of surface light fields
- Compression for optimal factored representations
  - PCA on SHRMs
- Theoretical analysis of sampling rates, resolutions
  - General framework for sampling in image-based rendering

Acknowledgements

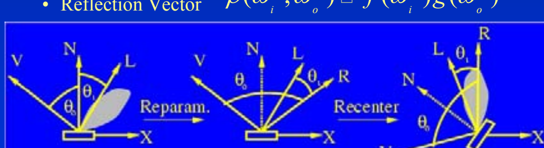
- Stanford Real-Time Programmable Shading System
  - Eric Chan, Bill Mark, Kekoa Proudfoot
- Readers of early drafts
  - Li-Yi Wei, Olaf Hall-Holt, anonymous reviewers
- Models
  - Armadillo: Venkat Krishnamurthy
  - Light probes: Paul Debevec
- Funding
  - Hodgson-Reed Stanford Graduate Fellowship
  - NSF ITR #0085864 “Interacting with the Visual World”

The End

BRDF Parameterization

BRDF

- Direct $\rho(\vec{\omega}_i, \vec{\omega}_o) \approx f(\vec{\omega}_i) g(\vec{\omega}_o)$
- Half Angle $\rho(\vec{\omega}_h, \vec{\omega}_i, \vec{\omega}_o) \approx f(\vec{\omega}_i) g(\vec{\omega}_h) f(\vec{\omega}_o)$
  [Rusinkiewicz 98, McCool et al. 01]
- Reflection Vector $\rho(\vec{\omega}_i^R, \vec{\omega}_o^R) \approx f(\vec{\omega}_i^R) g(\vec{\omega}_o^R)$



Parameterization

- Lighting: 2D function on a sphere $L(\vec{\omega}_i)$
- BRDF
  - Direct
  - Half Angle
  - ***Reflection Vector*** $\rho(\vec{\omega}_i^R, \vec{\omega}_o^R)$
- OLF
  - Direct
  - No Half Angle
  - ***Reflection Vector*** $B(\vec{R}, \vec{V})$

OLF Parameterization

- Direct $B(\vec{N},\vec{\omega}_o) \approx f(\vec{N})g(\vec{\omega}_o)$
- Reflection Vector (reflection, normal, view)
  - Captures structure of BRDF and OLF
  - Reflective BRDFs, OLFs become low-dimensional

$$B(\vec{R},\vec{N},\vec{V}) \approx f(\vec{R})g(\vec{N})h(\vec{V})$$

$B(\vec{R},\vec{N}) \approx f(\vec{R})g(\vec{N})$

Latta and Kolb 02
Wood et al. 00

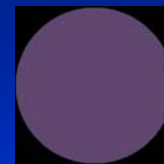
$B(\vec{R},\vec{V}) \approx f(\vec{R})h(\vec{V})$

Advantages
- Good param. for *both* BRDF, OLF
- Fast computation with convolution
- Single reflection map for each view

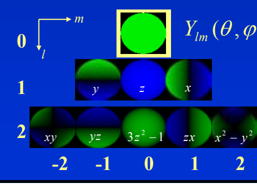
SHRM approximation

Exact Image

$B_R(\vec{V})$

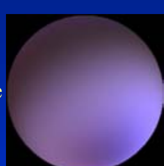


Order 0
1 term

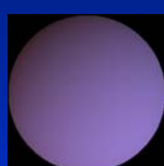


SHRM approximation

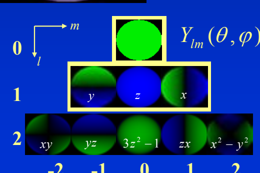
Exact Image



$B_R(\vec{V})$

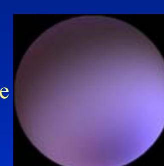


Order 1
4 terms

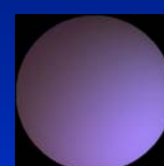


SHRM approximation

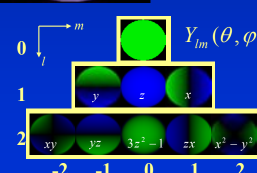
Exact Image



$B_R(\vec{V})$



Order 2
9 terms



Example: Phong BRDF

$$C_f = O(S^2\sqrt{s})$$ Frequency

Cost

$$C_a = O\left(S^4/s\right)$$ Angular

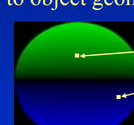
S = resolution, s = Phong exponent

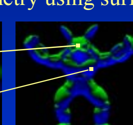
Frequency space faster unless s > 500

Usually 3 to 4 orders of magnitude faster (< 1 s compared to minutes or hours)

Orientation Light Field

- 4D function of surface normal, viewing direction
- Mapped to object geometry using surface normal





- Equivalent to image of sphere from all directions

Reflection Equation

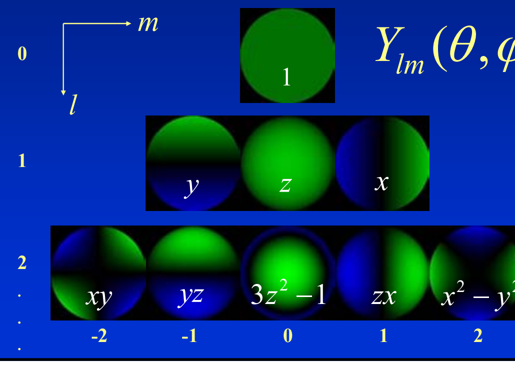
$$B(\vec{N},\vec{\omega}_o) = \int_{\Omega} L\left(R(\vec{N})\,\vec{\omega}_i\right)\rho\left(\vec{\omega}_i,\vec{\omega}_o\right)d\omega_i$$

Reflected Radiance (4D Orientation Light Field) — Distant Lighting (2D Environment Map) — Isotropic BRDF

$$B = L \otimes \rho$$

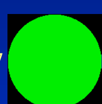
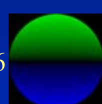
Basri and Jacobs 01
Ramamoorthi and Hanrahan 01

Spherical Harmonics



$$Y_{lm}(\theta,\varphi)$$

Spherical Harmonic expansion

 $= .67$  $+ .36$  $+ \ldots$

Expand Lighting, BRDF, OLF in spherical harmonics

$$L(\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{+l} L_{lm}Y_{lm}(\theta,\phi)$$

Convolution

- Lighting $L(\vec{\omega}_i)$ coefficients L_{lm}
- BRDF $\rho(\vec{\omega}_i^R,\vec{\omega}_o^R)$ $\rho_{lq,pq}$
- OLF $B(\vec{R},\vec{V})$ $B_{lm,pq}$

$$B = L \otimes \rho$$

$$B_{lm,pq} = L_{lm}\,\rho_{lq,pq}$$

Ramamoorthi and Hanrahan 01

This Session

Latta and Kolb: Homomorphic single-term factorization

- Advantages of SHRMS: more accurate, easier to analyze errors/set resolutions, fast computation using convolution
- Disadvantage: Multi-term, fixed parameterization.
- Future work: compute best single-term approximation, or other factorizations directly from SHRM using PCA

This Session

Sloan et al., Kautz et al: Low frequency lighting

Advantages of SHRMS
- General lighting environments, BRDFs
- Error analysis determines number of terms
- Rapid computation

Disadvantage: As yet, no shadows, interreflection

Results

- SHRM accuracy: comparisons with previous methods (Cabral et al. 99, Kautz and McCool 00) in paper
- Speed of prefiltering: speedups of 3 to 4 orders of magnitude; times in fractions of a second
- Real-time rendering even with multiple SHRMs

Video

Video

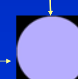
OLF Structure: Phong

Environment Map  →  Phong Reflection Map (blurred environment map)

$$B(\vec{R},\vec{V}) = B(\vec{R})$$

$B_V(\vec{R})$
2D view array of reflection maps

$B_R(\vec{V})$
2D image array of view maps

 → 

Same reflection map for all views

Different constant for each R

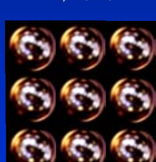
OLF Structure: Phong

$$B(\vec{R},\vec{V}) = B(\vec{R})$$

$B_V(\vec{R})$

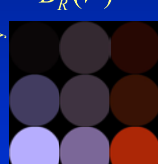
$B_R(\vec{V})$

View$_y$



View$_x$

Reflection$_y$



Reflection$_x$

Same reflection map for all views

Lumispheres constant for each R

Previous Work

Environment Maps
- Blinn & Newell 76, Miller & Hoffman 84, Greene 86, …
- Kautz & McCool 99, McCool et al. 01
- Cabral et al. 99
- Latta and Kolb 02

Frequency Space Methods (spherical harmonics)
- Cabral et al. 87, Sillion et al. 91, Westin et al. 92
- Ramamoorthi & Hanrahan 01
- Basri & Jacobs 01

OLF Factorization

$$B(\vec{R},\vec{N},\vec{V}) \approx f(\vec{R})g(\vec{N})h(\vec{V})$$

$$B(\vec{R},\vec{N}) \approx f(\vec{R})g(\vec{N})$$

Advantages

- Naturally captures diffuse, reflective parts

Latta and Kolb 02
Wood et al. 00

$$B(\vec{R},\vec{V}) \approx f(\vec{R})h(\vec{V})$$

Advantages

- Good param. for *both* BRDF, OLF
- Fast computation with convolution
- Single reflection map for each view